

Problem 1.9

[Difficulty: 2]

1.9 A cylindrical tank must be designed to contain 5 kg of compressed nitrogen at a pressure of 200 atm (gage) and 20°C. The design constraints are that the length must be twice the diameter and the wall thickness must be 0.5 cm. What are the external dimensions?

Given: Mass of nitrogen, and design constraints on tank dimensions.

Find: External dimensions.

Solution: Use given geometric data and nitrogen mass, with data from Table A.6.

The given or available data is: $M = 5 \cdot \text{kg}$

$$p = (200 + 1) \cdot \text{atm}$$

$$p = 20.4 \cdot \text{MPa}$$

$$T = (20 + 273) \cdot \text{K}$$

$$T = 293 \cdot \text{K}$$

$$R_{\text{N}_2} = 296.8 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}} \quad (\text{Table A.6})$$

The governing equation is the ideal gas equation

$$p = \rho \cdot R_{\text{N}_2} \cdot T \quad \text{and} \quad \rho = \frac{M}{V}$$

where V is the tank volume $V = \frac{\pi \cdot D^2}{4} \cdot L$ where $L = 2 \cdot D$

Combining these equations:

Hence
$$M = V \cdot \rho = \frac{p \cdot V}{R_{\text{N}_2} \cdot T} = \frac{p}{R_{\text{N}_2} \cdot T} \cdot \frac{\pi \cdot D^2}{4} \cdot L = \frac{p}{R_{\text{N}_2} \cdot T} \cdot \frac{\pi \cdot D^2}{4} \cdot 2 \cdot D = \frac{p \cdot \pi \cdot D^3}{2 \cdot R_{\text{N}_2} \cdot T}$$

Solving for D
$$D = \left(\frac{2 \cdot R_{\text{N}_2} \cdot T \cdot M}{p \cdot \pi} \right)^{\frac{1}{3}} \quad D = \left(\frac{2}{\pi} \times 296.8 \cdot \frac{\text{N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \times 293 \cdot \text{K} \times 5 \cdot \text{kg} \times \frac{1}{20.4 \times 10^6} \cdot \frac{\text{m}^2}{\text{N}} \right)^{\frac{1}{3}}$$

$$D = 0.239 \cdot \text{m}$$

$$L = 2 \cdot D$$

$$L = 0.477 \cdot \text{m}$$

These are internal dimensions; the external ones are 2 x 0.5 cm larger:

$$L = 0.249 \cdot \text{m} \quad D = 0.487 \cdot \text{m}$$